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Why lot? How sortition could help representative democracy

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ABSTRACT

In this paper we present a new analytical model of a Parliament and investigate the beneficial effects of the selection of legislators by lot in order to reduce some of the drawbacks of modern representative democracies. Resorting to sortition for the selection of public officers used to be in the past a popular way of taming factionalism in public affairs. Factionalism is assumed to be detrimental since public officers tend to favour their own faction instead of pursuing the general interest. In this respect our mathematical model shows in a rigorous way how it is possible to improve the efficiency of a Parliament by introducing the use of sortition to select part of its members. It will be shown that, starting from a Parliament working with two parties (or coalitions), where the costs of representative democracy are quite apparent through the detrimental effects of party discipline, one can beneficially move towards a Parliament where independent, randomly selected legislators sit alongside elected members. In particular, we show that increasing the number of independent legislators up to a critical point enhances the efficiency of the Parliament and puts into check the factionalism likely to arise from party discipline.

1. Introduction

The use of random numbers in Monte Carlo techniques was introduced long time ago by Ulam and Metropolis [1] and it is today a well-established and widely used tool in physics and other related disciplines in order to solve numerically very complicated problems [2]. More recently, the discovery of the beneficial effects of random noise in the phenomenon of the stochastic resonance, with so many applications in various fields [3,4], and the success of random strategies in Parrondo biased games [5,6], have stimulated several possible extensions of these strategies in the socio-economic context. Such extensions range from the adoption of random promotions, in order to circumvent the Peter Principle and increase the efficiency of hierarchical organizations [7,8], to the use of random investments in financial markets, aimed to diminish the risk of bubbles and increase the gain of traders [9–13]. Along this direction, in a previous paper [14] we proposed an agent-based model able to confirm the beneficial role of randomness also in the political context. In particular, we showed through extended numerical simulations that it is possible to improve the efficiency of a virtual Parliament by considering the introduction of a part of the legislators chosen by lot and independent of parties or coalitions.

The adoption of the random selection in politics, called 'sortition', has a long and very successful historical tradition which started at the beginning of democracy, in old Athens, and continued along the centuries up to the French

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Revolution [15,16]. Resorting on random procedures when it comes to grant powers, assign public functions or take collective decisions, has been interpreted in different ways in the recent literature. In some cases those procedures have been viewed as ways of contrasting rent-seeking behaviour [17] or fighting corruption [18,19]. In other cases, sortition has been looked as a way to accurately represent a diverse population in a smaller subset [20] or to make sure societies achieve allocative justice [21].

Sortition has also been considered as an effective instrument to fix some specific drawbacks of contemporary political systems and, in particular, to fight concentration of political power [22,23]. Some of the undesirable effects of political power, especially those currently more widespread, like corruption, elitism, self-referential behaviour, are often attributed to such a concentration. Viewed in this way, sortition parallels other mechanisms designed to keep political power in check, like division of powers, universal suffrage, political term limits, rotational assignment.

As a matter of fact, through history, sortition has done quite a good job in harnessing a specific source of power concentration, i.e. factionalism. By factionalism we mean a tendency to strengthen bonds within a group, usually in opposition to other groups, with the purpose of gaining more power. When such bonds are particularly strong, factionalism may lead to excessive concentration of power. If a faction holds a government office, it may use the benefits arising from that office to strengthen those bonds even more, with the purpose of fighting opposing factions. Eventually this may lead to relax the pursuance of the general interest [24].

The current parliamentary version of factionalism is party discipline. Party discipline is called for whenever internal cohesion is required to stress group identity against opposing parties [25,26]. Resuming lot as a way to contrast factionalism and to mitigate the detrimental effects of party discipline, in this paper we present a new model of a Parliament with a two-coalition party composition. The problem we are trying to address here is the same as in [14], i.e. whether the whole job of making decisions (the job of members of Parliament) can yield better results by changing the composition of Parliament, in particular by adding a third component of legislators selected by lot and independent of parties. However, the approach is different, since the new proposed model is simpler and more suited to an analytical approach.

The paper is organized as follows. The second section introduces the mathematical model and its main assumptions, also showing the differences with our previous study; the third section addresses the analytical calculation of the Parliament's efficiency in the two limiting cases of only two parties or only independent legislators. Section 4 presents the results for the most general case of a mixed Parliament, with both parties and independent legislators, while Section 5 looks for its maximum efficiency. Finally, in Section 6 we present a discussion with some conclusions.

2. Modelling a Parliament

In this section we present a one dimensional mathematical model of a Parliament with N members (legislators) A_i (i = 1,…,N) and two parties (or coalitions): the majority party (P_1) and the opposition (minority) party (P_2). From now on we will consider the terms "party" and "coalition" as synonymous, since our model applies indistinctly to both of them. As a matter of fact, two-party systems and two-coalition systems are two distinct models. In the former, two parties enter Parliament,[1] as in the case of the House of Commons and the United States Congress. In the latter, the different parties accessing Parliament gather together in two different coalitions, as in the case of the so-called "Second Republic" in Italy between 1996 and 2008. For the time being, we set the percentage of legislators of the two parties as: P_1 = 60% and P_2 = 40%.

During a given Legislature L, each legislator can perform only two actions: submitting a bill for approval and voting in favour or against any bill. Legislators are represented as points of a 1D space, i.e. of a horizontal axis indicated with the capital letter Y. Each point of this Y-axis, associated to a real number in the interval $[-1, +1]$, shows the social gain attached to the bill submitted by the legislator A_k. This represents the welfare threshold below which the legislator is not prepared to go when it comes to vote, therefore it needs to be chosen within a finite interval. In this way we have members who would be prepared to accept anything, however low may be the social contribution the decision is supposed to make, and members of Parliament who expect decisions to produce a significant contribution to social welfare before they can be prepared to vote in their favour.

As explained in the introduction, in Ref. [14] an agent-based model of Parliament has been proposed. In that model, members of Parliament lived in a 2D space, since two different thresholds were associated to each legislator, one related to a personal interest (X-axis) and another related to the social welfare (Y-axis). The simpler 1D version proposed here considers only the second threshold (maintaining the same name for the axis), but somehow accounts for the influence of the personal interest (as better explained later).

Within the 1D space, the distributions of legislators belonging to the two parties can be represented, in the limit for $N \gg 1$, as two probability density functions (PDF) defined over the Y-axis (see Fig. 1). Both the distributions $P_1(Y)$ and $P_2(Y)$ are assumed to be Gaussians, i.e. $P(Y) = (1/\sqrt{2\pi}\sigma)\exp[-(Y - \langle Y\rangle)^2/2\sigma^2]$, with means $\langle Y_1\rangle$ and $\langle Y_2\rangle$ and with the same standard deviation σ. Both the curves are normalized to have unitary area, but the size of the P_2 curve in this and in the next figures has been reduced only in order to distinguish it at a first sight.

[1] In actual fact, residual political forces sometimes enter a two-party or a two-coalition system. However, their real power can be considered as irrelevant.

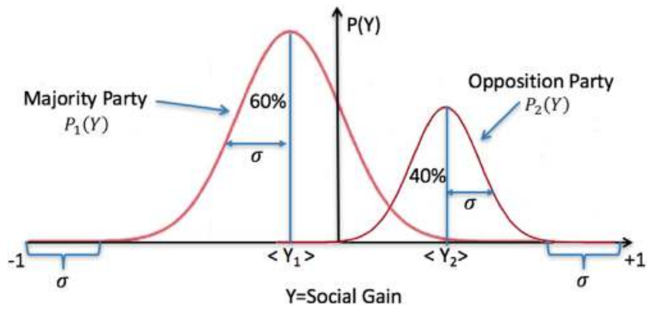

Fig. 1. Gaussian distributions of legislators belonging to the two parties $P_1(Y)$ and $P_2(Y)$. The two curves also represent the distributions of proposals coming from the two parties.

For a given legislative term L, the centroids $\langle Y_1 \rangle$ and $\langle Y_2 \rangle$ of the distributions are fixed and randomly chosen in the interval $[-1 + \sigma, 1 - \sigma]$. Of course this implies that, in some cases, the left or the right tail of a given distribution could overstep the limits of the interval $[-1, 1]$. In these cases we could think to either truncate the distribution or let it to exceed the limit, then considering only the points inside the interval. However, we will see later that our analytical treatment will be independent of this particular choice.

During each term (legislature), we assume that each legislator puts forward the same number of proposals. Therefore, the overall percentage of proposals coming from the members of a particular party is equal to the percentage of legislators of that party. In reality, regardless of the ratio between the number of legislators of the two parties, the members of the minority party have limited opportunities to propose legislation. However we assume for simplicity that each member of Parliament can propose the same number of bills. In other words, we assume that both majority and minority legislators are insensitive to the outcome of the vote, which means that they put forward legislation regardless of the likely result of the vote. Since the Y coordinate of each legislator represents the social gain of her proposal, the distributions $P_1(Y)$ and $P_2(Y)$ can be also regarded as the distributions of their proposals.

When it comes to voting a given proposal, say p^* with abscissa Y^*, party discipline applies for each legislator A_k. Party discipline implies that each legislator belonging to a party votes any proposal coming from a member of the same party regardless of the proposal social gain. Complying with such a discipline, and the consequent indifference to the social gain deriving from different proposals, can be interpreted as the cost that any elected legislator has to bear in exchange of the benefits deriving both from having been a successful candidate in the current term, and from the expected (and hoped for) candidacy in future terms.

Suppose A_k belongs to the majority party P_1 (but of course the same considerations hold also for P_2), there are two possibilities:

• p^* is an internal proposal (i.e. it comes from a member of P_1): it is accepted by A_k due to party discipline;

• p^* is an external proposal (i.e. it comes from a member of the other party P_2): it is accepted only if Y^* is greater than the party mean $\langle Y_1 \rangle$, which represents the *minimum* social gain which a proposal coming from legislators of P_2 should yield to be accepted by legislators of P_1; moreover, if this condition is fulfilled, due to internal motivations (personal interests) of P_1, A_k will vote only a fraction equal to *one half* of all the proposals. Indeed, when proposals come from P_2, the party discipline is no longer strictly applicable, which means that majority party legislators will feel free to vote according even to their own interest, provided that the party threshold is respected.

Proposals are accepted by Parliament provided they receive half plus one ($N/2 + 1$) of the votes. Due to party discipline for internal proposals, this requirement will be always fulfilled for the majority party P_1, which alone has 60 per cent of the legislators. We should not forget, however, the proposals of the opposition party. The final number of accepted proposals at the end of a term will depend also on the contribution of the opposition Party P_2, whose proposals are voted inasmuch as the majority party likes them. Therefore, the number of opposition party proposals which finally gets to be approved should also depend on the relative position of the two parties along the Y axis.

Actually, due to the influence of personal interests, only one half of members of P_1 will vote for the opposition proposals lying on the right of the P_1 mean. This means that, for these proposals (indicated by the dark parts of the P_2 Gaussians in Fig. 2), 30% of legislators, belonging to P_1, will sum their vote to the 40% of legislators entirely belonging to P_2, thus always exceeding the $N/2 + 1$ threshold necessary for the approval of the proposals.

In this paper we will always consider an ensemble of N_L parliamentary terms, each one with a different random position of the centroids of both parties. Averaging over many terms ($N_L \gg 1$), the asymptotic distributions of both parties will be centred at $Y = 0$ (see Fig. 3). This is because, across a very large number of terms, the central position of each Gaussian distribution has the same probability of occurring. Therefore, the expected mean value of these positions is 0 given that the horizontal axis spans from -1 to 1. Furthermore, since we are considering a model in logical time and not in historical time, "surprises" such as new policy issues are ruled out.

In the following we will always make the assumption of an infinite numbers of terms for all the analytic derivations. This is also the reason why the fraction of legislators of a given party, say P_1, accepting proposals coming from P_2,

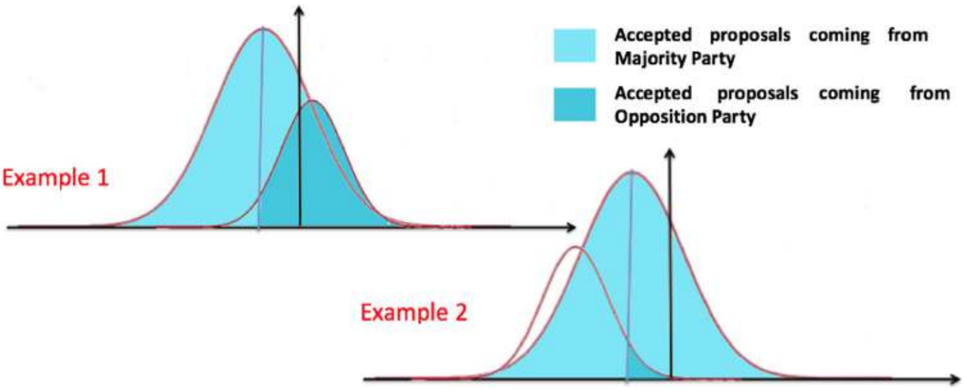

Fig. 2. A couple of example where the fraction of accepted proposals coming from P_2 (in dark blue) are shown as function of the relative positions of the two parties.

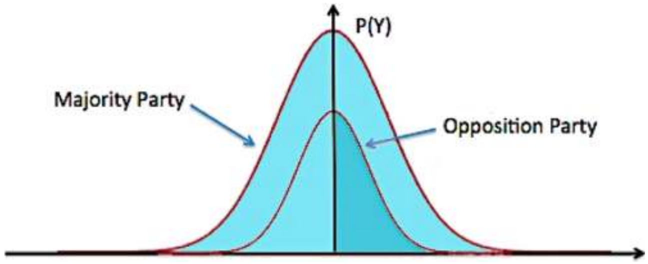

Fig. 3. Asymptotic distributions of both parties after averaging over many terms ($N_L \gg 1$). The contribution of the opposition party P_2 to the expected percentage of accepted proposals is now represented by the *positive* side (in dark blue) of the corresponding asymptotic distribution.

corresponds to 50% of the members of P_1(failing personal interest it would be 100%). Moreover, under this hypothesis, the opposition party proposals, which finally get to be approved with the contribution of the majority, will be all the positive ones. We will analyse in detail this situation in the next section.

Let us define, now, the global efficiency *Eff* (L) of a term. It will be given by the net social gain yielded by the accepted proposals, i.e. by the product of the percentage $N_{\%ACC}$ (L) of the accepted proposals (calculated with respect to the total number of proposals put forward during the term) times their average social gain Y_{AV} (L):

$$Eff\ (L) = N_{\%ACC}\ (L) \cdot Y_{AV}(L) \tag{1}$$

Averaging over N_L terms, one can obtain the expected global efficiency:

$$Eff_{exp} = \frac{1}{N_L} \sum_{j=1}^{N_L} Eff(L) \tag{2}$$

Defining $N_{\%ACC}$ as the expected percentage of accepted proposals and Y_{AV} as their expected social gain, both averaged over the N_L terms:

$$N_{\%ACC} = \frac{1}{N_L} \sum_{j=1}^{N_L} N_{\%ACC}\ (L) \qquad Y_{AV} = \frac{1}{N_L} \sum_{j=1}^{N_L} Y_{AV}(L)$$

one can also expect that, in the limit $N_L \gg 1$:

$$Eff_{exp} \cong N_{\%ACC} \cdot Y_{AV} \tag{3}$$

This definition is the simplest way to account simultaneously for two measures uncorrelated with each other: Parliament output and quality of approved proposals. The expected global efficiency is the fundamental measure of the efficiency of a deliberative body like a Parliament. In the following we shall develop this notion much further with the purpose of investigating the effects on such a measure of a variable composition of Parliament. In particular, we shall address the question of what would happen if a variable percentage of members of Parliament had no longer an obligation of following the party line, simply because they do not belong to any party. For this reason, these legislators will be called "independent". At variance with legislators elected as members of a party,[2] independent legislators might be assumed to be drawn randomly from the population of constituents. The whole purpose of this paper is precisely to show how the expected global efficiency changes as Parliament allows a given number N_{ind} of randomly selected independent members in, free of any party discipline.

In so doing, we are assuming that the randomly drawn citizens are as skilled as the elected ones. One could object that politicians are usually more experienced than ordinary citizens. This is mostly true. However, bear in mind that we are taking into account legislators and not governors. Legislators are people who are mainly assumed to be representative of other people, whereas governors are people who are mainly committed to the execution of specific policies, where particular skills are needed. Furthermore, Parliaments are usually provided with all sort of offices, with the technical knowledge needed for specific issues.

3. Expected global efficiency for the two polar cases: $N_{ind} = 0$ and $N_{ind} = N$

In this section we will show how to derive analytically the proposed measure of parliamentary efficiency in the limit of many legislators ($N \gg 1$) and many parliamentary terms ($N_L \gg 1$). These limits will allow us to consider Gaussian distributions of legislators and to substitute integrals to the summations for calculating the averages of the various quantities. For the time being, the derivation will concern the two polar cases: a Parliament with just *two parties*, i.e. with $N_{ind} = 0$ independent legislators, and a Parliament with just randomly selected members, i.e. with $N_{ind} = N$.

A Parliament with only two parties (60%–40%)

To determine the global efficiency in the case of a Parliament with a majority party P_1 with 60 per cent of members and an opposition party P_2 with 40 per cent, we proceed by evaluating separately the two factors in Eq. (3). In accordance with the assumption $N_L \gg 1$, for the Gaussian distributions of the two parties we assume that $\langle Y_1 \rangle = \langle Y_2 \rangle = 0$.

This assumption parallels that of the previous section according to which, due to the large number of terms considered, the random positions of the legislators lead on average to a 0 social gain. The same argument is now extended to the minority party, and leads, in average, to the same result, even if for any single term the two parties are likely to have two different centres of distribution. This is not surprising as the two parties differ not for the centre of the distribution but for the different role they play in Parliament, as majority and minority party.

It is worth noticing that the assumption $\langle Y_1 \rangle = \langle Y_2 \rangle = 0$, together with the choice of a standard deviation always fixed at the value $\sigma = 0.15$, will also allow us to overcome the apparent problem of the Gaussian's tails sometimes exceeding the limits of the definition interval $[-1, 1]$. In fact, under these conditions (which will be always adopted from now on), the tails of any average distributions of the form $P(Y) = (1/\sqrt{2\pi}\sigma)\exp(-Y^2/2\sigma^2)$ will be included, with an excellent approximation, within the considered interval, making our analytical derivations correct in the limit $N_L \gg 1$.

Because of the previously illustrated voting rules and recalling that the percentage of proposals coming from the members of a party is equal to the percentage of legislators of that party, the expected percentage of accepted proposals $N_{\%ACC}$, averaged over many parliamentary terms, is given by the sum of two elements (consider again Fig. 3 as reference): the first one, which is the contribution of party P_1, is represented by the *whole* area below the corresponding asymptotic distribution $P_1(Y)$, while the second one, the contribution of party P_2, is represented only by the (dark) area below the *positive* part of the corresponding asymptotic distribution $P_2(Y)$, since – as already shown in the previous section – the positive proposals coming from the opposition party are the only ones voted also by the majority party.

Here follows the formal expression:

$$N_{\%ACC} = \left[\int_{-1}^{1} P_1(Y)\, dY\right] \cdot 60\% + \left[\int_{0}^{1} P_2(Y)\, dY\right] \cdot 40\%$$

where the two contributions are weighted according to the percentage of proposals put forward by each party, in this case equal to its relative size. Since both the distributions are normalized, i.e. their total area is equal to 1, the expected value for $N_{\%ACC}$ over the entire set of N_L terms will be:

$$N_{\%ACC} = 1 \cdot 60\% + 0.5 \cdot 40\% = 80\%$$

In the same fashion, the expected average social gain Y_{AV} of these proposals over the same set of parliamentary terms is given, again, by the sum of two elements, stemming from the contributions of P_1 and P_2 (in terms of fraction of proposals submitted multiplied by their welfare contribution)

$$Y_{AV} = \left[\int_{-1}^{1} P_1(Y)\, YdY\right] + \left[\int_{0}^{1} P_2(Y)\, YdY\right]$$

The first integral gives a null result since, because of party discipline, the majority party accepts all its internal proposals, regardless of their contribution (positive or negative) to the social gain; on the other hand, the second integral

[2] As a matter of fact no electoral system is explicitly assumed in the paper. The particular distribution of party members however is an indication that those members of Parliament are not randomly selected.

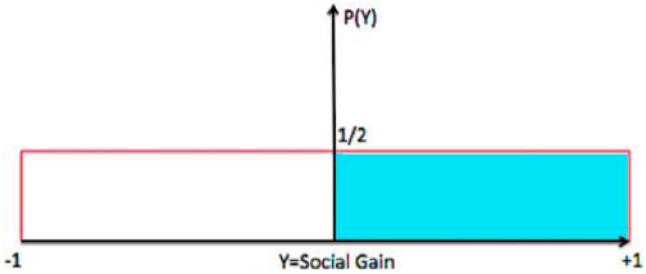

Fig. 4. Uniform distribution of independent legislators.

is calculated only over the positive Y-axis, therefore we expect that it will give a small positive result. Its calculation is straightforward and gives the approximated result 0.06.

In conclusion, the expected average social gain will be:

$$Y_{AV} \approx 0 + 0.06 = 0.06$$

Therefore, the expected global efficiency over many parliamentary terms will be:

$$Eff_{exp} = N_{\%ACC} \cdot Y_{AV} = 80\,(\%) \cdot 0.06 = 4.8\% \tag{4}$$

It is not surprising that we get such a very small value. The only positive contribution to social welfare comes from the opposition party that, thanks to the support of the majority party, will see its positive proposals approved with a large majority. It may sound paradoxical that the majority party is capable of giving rise to a positive social gain only when other parties' proposals are at stake. We will see later that this value can be increased if different circumstances apply. In particular, we shall look at the possibility of filling Parliament with independent legislators, free from any party linkage.

This could be realized in practice by selecting them at random from all the citizens with the necessary requirements (in principle, the same that allow them to express their preferences for the parties during the elections). Because of this particular selection procedure, we will assume that the independent legislators are not subject to any kind of party discipline: each of them votes independently from the other independent legislators and from the parties. In particular, given a proposal p^* with abscissa Y^*, any independent member A_k with abscissa Y_k will accept it only if $Y^* > Y_k$, since Y_k represents the *minimum* social gain that the proposal should yield for it to be accepted by A_k. Again, as already seen for the parties' vote, for a given proposal p^*, we assume that, because of internal motivations (*personal interests*), only 50% of the independent legislators fulfilling the condition $Y_k < Y^*$ will accept the proposal.

A Parliament with only independent legislators

Let us now consider the extreme case of an entirely independent Parliament, that is a Parliament with only independent legislators ($N_{\text{ind}}= N$). We assume that, in the limit $N \gg 1$, their probability distribution P(Y) along the Y-axis is a uniform one (from -1 to $+1$) with unitary area (see Fig. 4). This is because the members of a party, and therefore the elected legislators, are usually more politically homogeneous than independent legislators. Indeed, they are elected through the formation of electoral rolls, which are made of individuals not randomly selected but selected according to their degree of political homogeneity. It is therefore reasonable to assume that homogeneous political propensities do imply similar social gain thresholds. We convey the difference between homogeneity of political thresholds coming from members of the same party, and non-homogeneity of those deriving from independents by adopting an uniform distribution for the latter. Due to this requirement, we will have $P(Y) = 1/2$.

We know that the main feature of independent legislators is that they are not subjected to any party discipline. Consequently, they will vote according to their own free convictions. However, in such circumstances (since an average over many parliamentary terms is considered), none of the proposals will be accepted. In fact, it is quite clear that only proposals with a positive abscissa Y^* could have a chance to be accepted, since the required majority could be reached only for those proposals (see dark area in Fig. 4). However, because of their personal interests, we know that only one half of the independent legislators with $Y_k < Y^*$ will accept these proposals. Therefore, to reach $N/2 + 1$ votes, a given proposal should have an abscissa Y^* not only greater than zero but also greater than 1.

In conclusion, for a hypothetical Parliament with only independent legislators, we will always find a null result for the expected values of both the percentage of accepted proposals and the average social gain. This means that the expected global efficiency over many terms will be:

$$Eff_{exp} = N_{\%ACC} \cdot Y_{AV} = 0 \tag{5}$$

This unexpected result makes party discipline not so difficult to accept, especially when it makes good proposals (i.e. with a positive value of social gain) easier to accept. But certainly not when it is designed to impose the dictatorship

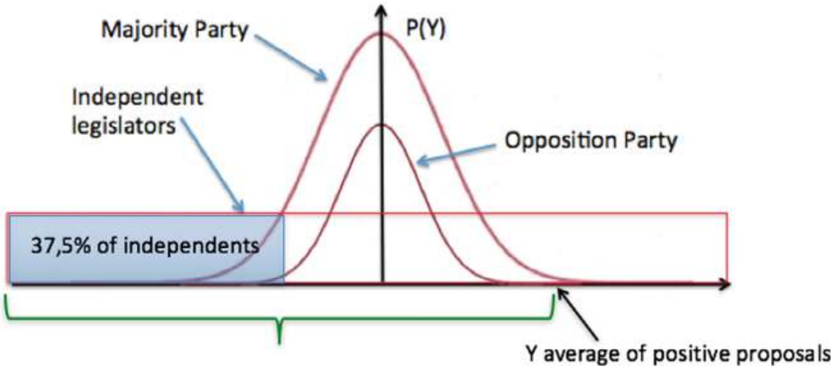

Fig. 5. Percentage of independent legislators voting, on average and in the limit of many legislative terms, for the positive proposals coming from the parties.

of the majority. As argued above, in a Parliament without parties, no proposal has enough votes to get approved, since no proposal is good enough for half of the legislators. This result mirrors that arising from a Parliament with only parties, where hardly any positive contribution to social welfare is likely on average to emerge.

However, it has certainly not gone unnoticed that the very small efficiency associated to both illustrated cases – a Parliament with only parties and one with only independent legislators – depends on opposite reasons. In the case of a Parliament with parties many proposals get accepted, but they yield a very small average social gain. In the case of a Parliament with only independent legislators, Parliament accepts only extremely good proposals, but their number is close to zero. Therefore, we can expect that contaminating a Parliament with two parties with an increasing number of independent legislators would reduce the number of accepted proposals while increasing their average social gain (at least insofar as the percentage of accepted proposals is greater than zero). As a consequence, the value of the global efficiency as a function of the percentage of independents sneaking into Parliament, calculated as product point by point of the previous two quantities, should show an initial increase from its quite low extreme value typical of the two-party case, then it should reach a central peak in correspondence of a given percentage of uniformly distributed independents, and finally it should slip towards zero, when the percentage of independents approaches a hundred per cent. In the following section we will provide a demonstration that this is roughly the case.

4. A Parliament with an increasing number of independent members

Let us start with the analytic derivation of the percentage of accepted proposals as function of the number of independents, in a Parliament with $N \gg 1$ members and in the limit of many legislative terms. Just as an example, we will consider a Parliament with $N = 500$ members, but of course our results are valid for any (great) value of N. It is worth noticing that, with N_{ind} independent legislators, the real number of members belonging to the two parties, P_1 and P_2, has to be calculated by taking, respectively, 60 per cent and 40 per cent of the difference $N - N_{\text{ind}}$. For small values of N_{ind}, the 60 per cent of such a difference still represents the majority of the members of Parliament. For example, if $N_{\text{ind}} = 10$, then the 60 per cent of $N - N_{\text{ind}}$, i.e. 294, continues to represent the absolute majority of the members of Parliament. For increasing values of N_{ind}, the members of P_1 will be reduced. This implies in turn that, above a given threshold of N_{ind}, P_1 will no longer be the *absolute* majority party but only the *relative* majority one, that is the party with the biggest number of members, who are not, however, more than one half of the whole. We will see that it is precisely this feature, together with the role of the independent legislators, that makes enhancing the efficiency of the Parliament possible.

As we know, in the limit of many legislative terms, both parties are centred at $Y = 0$, thus positive proposals (i.e. proposals with $Y^* > 0$), which represent the 50% of the total number, are accepted by 100 per cent of the members of the party proposing them (through party discipline) and by 50 per cent of the members of the other party (due to the influence of personal interests). Furthermore, in principle, these proposals should be accepted also by one half of the independent legislators with abscissa $Y < 0$. However, from the independent legislators point of view, the average social gain of the positive proposals is not $Y = 0$ but $Y = 0.5$ (the middle point of the positive part of the Y axis). This means that some proposals, lying on the positive axis, will be voted by independent legislators having an abscissa higher than zero. Therefore, since the independent members with $-1 < Y < 0.5$ are the 75% of the total, for the dynamical rules of the model only one half of them will vote, on average, for positive proposals: thus, since they are uniformly distributed along the Y axis, a percentage of 37,5% of independent legislators will accept these proposals (as shown in Fig. 5).

Pooling together the three contributions of the majority party, the opposition party and independent legislators, we can easily work out the number of votes received, on average, by the positive proposals coming from the legislators, as a function of the percentage of independent legislators sitting in Parliament. The results are shown in Fig. 6. Notice

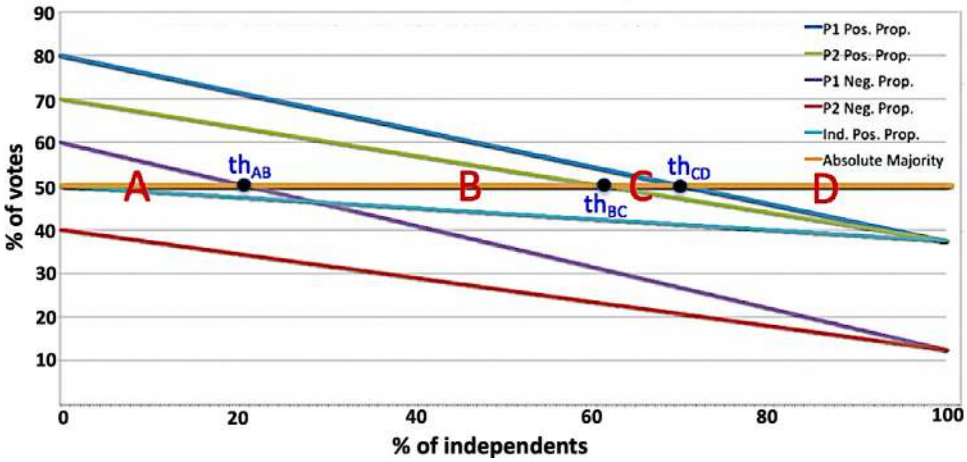

Fig. 6. Percentage of votes received, on average, by both the positive and the negative proposals coming from the three component of the Parliament, as function of the percentage of independent legislators. The absolute majority level is also reported as a horizontal line. Along this line, three thresholds are visible, under which the corresponding proposals do not reach enough votes to get passed.

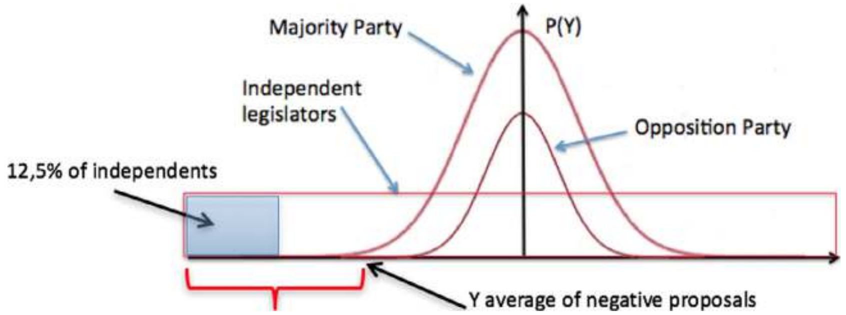

Fig. 7. Percentage of independent legislators voting, on average and in the limit of many legislative terms, for the negative proposals coming from the parties.

that, of course, in the two extreme cases on the x-axis, corresponding to 0% of independents (only parties) and to 100% of independents (no parties), we will not have any vote or proposal coming, respectively, from either independent legislators or members of parties.

Looking at the details of the plot, where the example of a Parliament with $N = 500$ members is considered, one can observe that:

- positive proposals coming from independent legislators are never accepted, since they never reach the threshold of $N/2 + 1 = 251$ votes (as shown with the help of the horizontal line, representing the absolute majority which corresponds to 50% of votes plus one);
- positive proposals coming from the relative minority party P_2 are accepted until the percentage of independent legislators in the Parliament stays below 61 per cent;
- positive proposals coming from the relative majority party P_1 are accepted until the percentage of independent legislators in the Parliament stays below 70 per cent.

In the same Figure, negative proposals have been also considered (i.e. proposals with $Y^* < 0$). It is straightforward to notice (and it is confirmed by the plot) that only those coming from the relative majority party P_1 can be accepted, as long as this party maintains the absolute majority, since its members accept both good and bad proposals, because of party discipline. Negative proposals coming from P_2 and independent legislators are never accepted. When P_1 loses the absolute majority, its negative proposals (on average) can still be accepted due to the contribution of one half of independent legislators with $-1 < Y < -0.5$ (who represent the 12,5% of the total, see Fig. 7), being $Y = -0.5$ the average social gain of negative proposals coming from P_1. This happens until the percentage of independent legislators in the Parliament stays below 21 per cent. It is interesting to notice that neither positive nor negative proposals coming from independent legislators can be accepted, no matter how many of them sit in Parliament.

Summarizing, in Fig. 6 we saw that three progressive thresholds th_{AB}, th_{BC} and th_{CD} do exist, identifying four different intervals in the percentage of independent legislators, namely A, B, C and D, each one with a given (decreasing) percentage

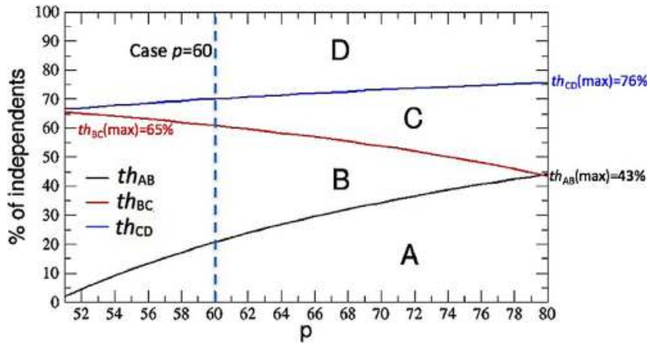

Fig. 8. Behaviour of the three thresholds th_{AB}, th_{BC}, th_{CD} as function of p for $N = 500$. The particular case of $p = 60\%$ is reported as a dashed vertical line.

of accepted proposals. Quite clearly, the values of these thresholds strictly depend on the size p of the relative majority party P_1. In the case considered, i.e. for $p = 60$ per cent, we already found (empirically) $th_{AB} = 21\%$ $th_{BC} = 61\%$ and $th_{CD} = 70\%$. The expressions of these thresholds for any value of p and any value of N can be easily determined in percentage by imposing the corresponding conditions on the number of votes and are the following:

$$th_{AB} = \frac{N(p-50)-100}{(p-12.5)}\frac{100}{N} \qquad th_{BC} = \frac{N(100-p)-200}{(125-p)}\frac{100}{N} \qquad th_{CD} = \frac{Np-200}{(p+25)}\frac{100}{N} \tag{6}$$

As expected, for $N = 500$ and $p = 60$ we recover, respectively, $th_{AB} = 21\%$ (corresponding to $N_{ind} = 103$), $th_{BC} = 61\%$ (corresponding to $N_{ind} = 305$) and $th_{BC} = 70\%$ (corresponding to $N_{ind} = 350$).

Having a look at the behaviour of th_{AB}, th_{BC}, th_{CD} as function of p for $N = 500$, reported in Fig. 8, the four regions A, B, C and D are well defined in a plausible range of values of p, going from 51% to 80%. On the other hand, for $p > 80\%$ the region B would disappear, but the values of p would be unrealistic: in fact, also with randomly selected legislators, the relative size of the two parties in the real world would continue to be decided by elections results, and it is very unlikely that any party or coalition obtains more than 60%–65% of preferences.

At this point, within each one of these four regions, we can analytically work out, for given values of p and N and in the limit of many legislative terms, both the percentage of accepted proposals $N_{\%ACC}$ and the average value of the social gain Y_{AV} produced by these proposals, as a function of the number of independent legislators N_{ind}. After some algebra, one obtains the following expressions:

Region A:

$$\begin{aligned} N_{\%ACC-A}(N_{ind}) &= p\frac{N-N_{ind}}{N}\int_0^1 P_1(Y)\,dY + (100-p)\frac{N-N_{ind}}{N}\int_0^1 P_2(Y)\,dY + +p\frac{N-N_{ind}}{N}\int_{-1}^0 P_1(Y)\,dY \\ Y_{AV-A}(N_{ind}) &= \frac{N-N_{ind}}{N}\left[\int_{-1}^1 P_1(Y)\,Y\,dY + \int_0^1 P_2(Y)\,Y\,dY\right] + \frac{N_{ind}}{N}0.5 \end{aligned} \tag{7}$$

Region B:

$$\begin{aligned} N_{\%ACC-B}(N_{ind}) &= p\frac{N-N_{ind}}{N}\int_0^1 P_1(Y)\,dY + (100-p)\frac{N-N_{ind}}{N}\int_0^1 P_2(Y)\,dY \\ Y_{AV-B}(N_{ind}) &= \frac{N-N_{ind}}{N}\left[\int_0^1 P_1(Y)\,Y\,dY + \int_0^1 P_2(Y)\,Y\,dY\right] + \frac{N_{ind}}{N}0.5 \end{aligned} \tag{8}$$

Region C:

$$\begin{aligned} N_{\%ACC-C}(N_{ind}) &= p\frac{N-N_{ind}}{N}\int_0^1 P_1(Y)\,dY \\ Y_{AV-C}(N_{ind}) &= \frac{N-N_{ind}}{N}\left[\int_0^1 P_1(Y)\,Y\,dY\right] + \frac{N_{ind}}{N}0.5 \end{aligned} \tag{9}$$

Region D:

$$\begin{aligned} N_{\%ACC-D}(N_{ind}) &= 0 \\ Y_{AV-D}(N_{ind}) &= 0 \end{aligned} \tag{10}$$

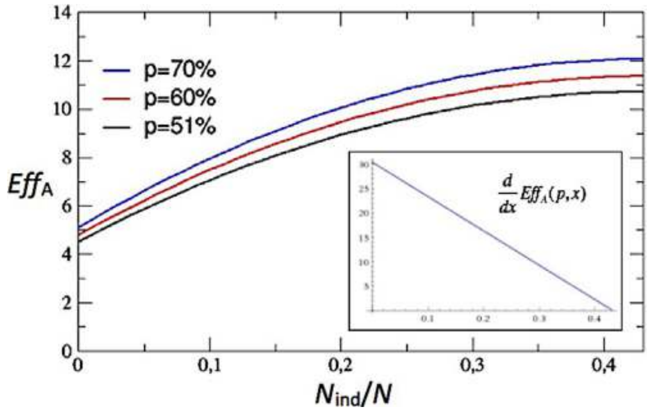

Fig. 9. Behaviour of the efficiency Eff_A as function of N_{ind}/N for three increasing values of p. The derivative of Eff_A is plotted in the inset and it is always positive in the same interval for any p.

5. Searching for the maximum efficiency

We are now ready to calculate the expected average global efficiency Eff_{exp} of our Parliament (in the limit of many legislative terms) as function of p, N and N_{ind}, in the four regions A, B, C and D, by multiplying inside each of them the value of $N_{\%ACC}$ for the corresponding value of Y_{AV}. Substituting in the expressions (7)–(10) the values of the definite integrals, which do not depend on p, N and N_{ind}, after some simple algebra we finally have the efficiency for each of the four regions:

$$Eff_A\,(p,N,N_{ind}) = \left(1-\frac{N_{ind}}{N}\right)\left(50+\frac{p}{2}\right)\left(0.06+0.44\frac{N_{ind}}{N}\right) \tag{11}$$

$$Eff_B\,(N,N_{ind}) = 50\left(1-\frac{N_{ind}}{N}\right)\left(0.12+0.38\frac{N_{ind}}{N}\right) \tag{12}$$

$$Eff_C\,(p,N,N_{ind}) = \left(1-\frac{N_{ind}}{N}\right)\frac{p}{2}\left(0.06+0.44\frac{N_{ind}}{N}\right) \tag{13}$$

$$Eff_D = N_{\%ACC-D}\cdot Y_{AV-D} = 0 \tag{14}$$

Notice that Eff_B does not depend on p while, as expected, Eff_D is always zero.

Let us present, here, the plots of these efficiency functions into each region. This exercise is crucial to support the main analytical conclusion of this work, i.e. the evidence that a mixed composition of Parliament is superior to any other option. The outcome of this exercise is not straightforward, but it is highly significant. The shape of the efficiency function is not the same as we move from a low p to a large one. However, in all cases a maximum exists for all sensible values of p. That maximum is unambiguously found in the area of a mixed Parliament. It is always a good idea to fill deliberative bodies with a limited number of randomly drawn members.

In Fig. 9 we plot the efficiency Eff_A as function of N_{ind}/N for three increasing values of p from 51% to 70%. We observe that, within its maximum possible range (i.e. between 0 and $th_{AB}(\max) = 43\%$, i.e. for $0 < N_{ind}/N < 0.43$), it is a monotonically increasing function (as also confirmed by the derivative plot in the inset):

In Fig. 10 we plot the efficiency Eff_B as function of N_{ind}/N, which does not depend on p. We observe that, in its maximum possible range (i.e. between 0 and $th_{BC}(\max) = 65\%$), it has always a maximum.

The value of the maximum ($N_{ind} = 34\%$) ensures that it remains in region B until the size p of the majority party stays below 70% (above this size, as visible in Fig. 8, region B becomes too small).

Plotting in Fig. 11 the efficiency Eff_C as function of N_{ind}/N for the same three values of p considered in Fig. 10, we notice that, in its maximum possible range (i.e. between $th_{AB}(\max) = 43\%$ and $th_{CD}(\max) = 76\%$), it is a monotonically decreasing function, as also confirmed by its derivative plotted in the inset.

Summing up so far, the behaviour of the average global efficiency in the three regions A, B and C, where it assumes non null values, seems consistent with the hypothesis that this efficiency has a minimum at the two extrema ($N_{ind} = 0$ and $N_{ind} = N$), starts to monotonically increase for small values of N_{ind} (region A), reaches a maximum for $N_{ind}/N = 0.34$ (region B), then monotonically decreases towards zero (region C). From there on, in region D, the efficiency Eff_D remains equal to zero.

Let us now look into this scenario in greater detail, and plot in Fig. 12 the average global efficiency in the range $0 < N_{ind} < th_{CD}(\max)$ for four increasing values of p. The positions of the three thresholds, th_{AB}, th_{BC} and th_{CD}, which separate the four regions, are indicated by vertical dashed lines. Of course, the sudden change in efficiency observed in all the plots

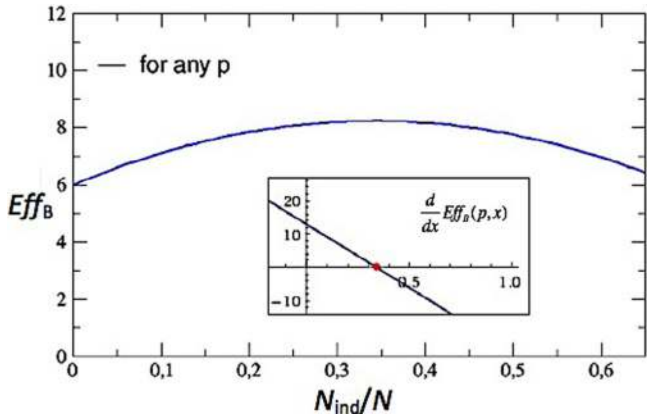

Fig. 10. Behaviour of the efficiency Eff_{B} as function of N_{ind}/N for any value of p. The derivative of Eff_{B} is plotted in the inset and the position of the maximum is visible.

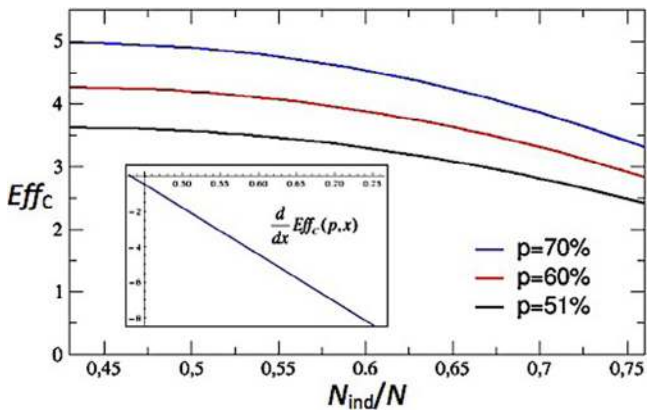

Fig. 11. Behaviour of the efficiency Eff_{C} as function of N_{ind}/N for three increasing values of p. The derivative of Eff_{C} is also plotted in the inset and it is always negative in the same interval for any p.

when the value of N_{ind}/N crosses each one of the three thresholds, is an effect of the assumed limit of many (infinite) legislative terms. Averaging over a relatively small number of them, the fluctuations mainly due to the random positions of the two parties along the Y-axis would make these transitions much smoother.

By looking closely at the four panels, it clearly appears that the position of the maximum value for the global efficiency, say Eff_{max}, is not always situated in region B but strictly depends on the value of p. As is visible in panels (a) and (b), the initial insertion of independent legislators in a Parliament with only parties induces a sudden increase in the global efficiency, similar for any value of p, which reaches its maximum value $Eff_{\text{max}}(\text{A})$ at the threshold th_{AB}. However, since the position of this threshold does depend on p, for intermediate values of N_{ind} it may occur that, as shown in panels (c) and (d), the value $Eff_{\text{max}}(\text{A})$ exceeds the maximum value of the efficiency in region B, $Eff_{\text{max}}(\text{B})$.

Following these insights, in Fig. 13 we plot, as dashed lines, both the constant position of the absolute maximum for $Eff_{\text{max}}(\text{B})$ (34% of independent legislators) and the variable position of the threshold th_{AB}, also expressed as percentage of the independent legislators. Then, in bold, we highlight the position of the global maximum efficiency $Eff_{\text{max}}(p)$. We found that, until $p < 56.5$ (%), it results $Eff_{\text{max}}(\text{B}) = 8.22 > Eff_{\text{max}}(\text{A})$, therefore $Eff_{\text{max}}(p) = Eff_{\text{max}}(\text{B})$. However, for p > 56.5 (%), $Eff_{\text{max}}(\text{A})$ starts to exceed 8.22, thus becoming the new global maximum efficiency $Eff_{\text{max}}(p)$. This implies that at $p = 56.5$ (%) the percentage of independent legislators needed to get the maximum efficiency suddenly rushes down from 34% to 14.3%, then it slowly goes back towards 34%, reached again around $p = 70$ (%), where region B tends to disappear and the maximum efficiency reaches its highest value $Eff_{\text{max}}\,(\text{A}) = 11.8$.

In conclusion, the analysis of the Parliament in the limit of many legislative terms confirms that an intermediate percentage $N_{\%\text{ind}}$ (between 14% and 34%) of legislators independent from the two parties and not subject to any party discipline, can always improve the efficiency of the system, regardless of the size of the relative majority party. This is the most fundamental result of this work. We have shown that, for realistic values of p, maximum efficiency of Parliament is not reached with a purely party-based Parliament nor with an entirely randomly based one, but with a sensible mixture of the two.

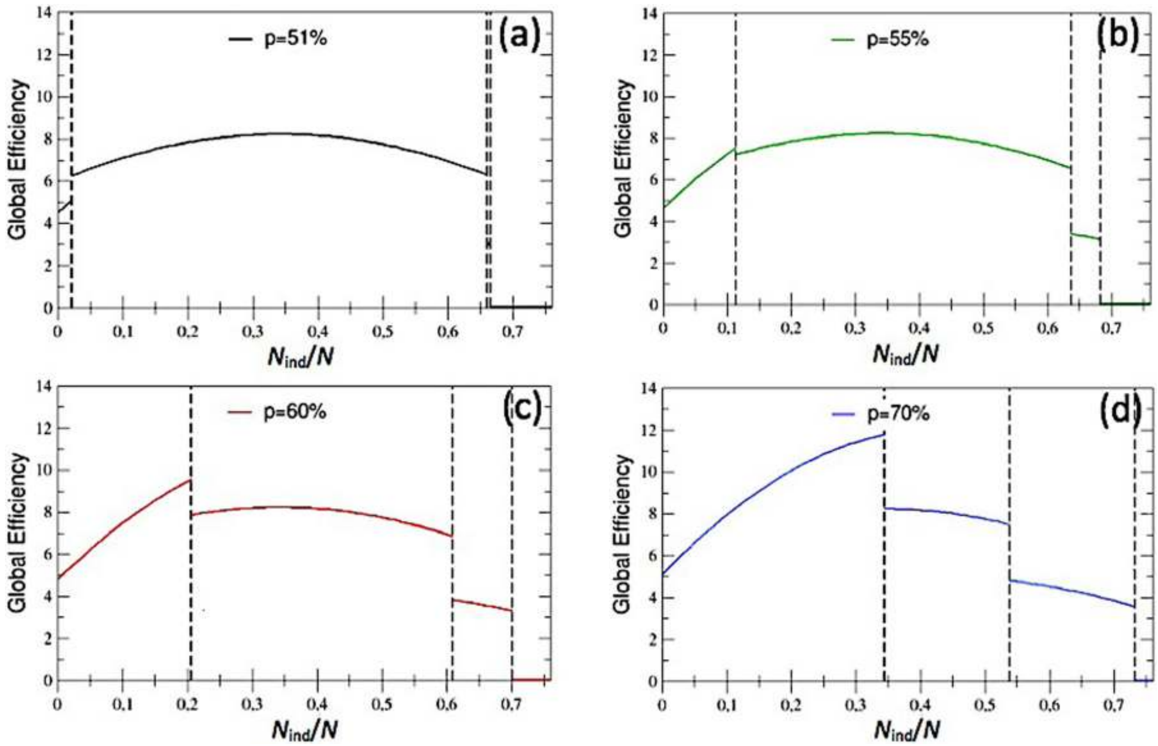

Fig. 12. The average global efficiency in the range $0 < N_{\text{ind}} < th_{\text{CD}}(\text{max})$ is plotted for four increasing values of p, i.e.: 51% (a), 55% (b), 60% (c) and 70% (d).

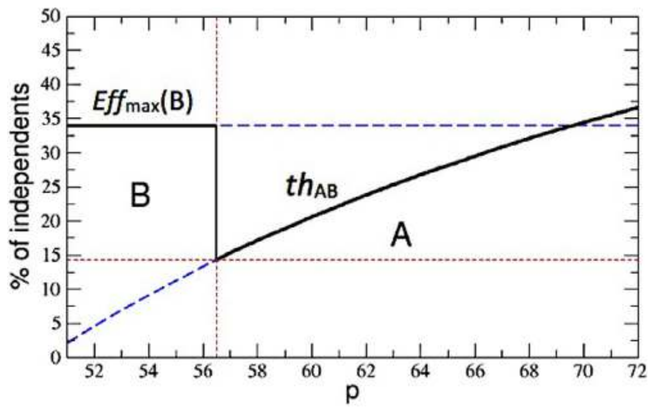

Fig. 13. The position of the absolute maximum for $Eff_{\max}(\text{B})$ and the variable position of the threshold $th_{\text{AB}} = Eff_{\max}(\text{A})$ are plotted as dashed lines. In bold, partially superimposed to the previous lines, we indicate the position of the global maximum efficiency $Eff_{\max}(p)$.

6. Discussion and conclusions

Representative democracy, when based on the party system, may fail to deliver its most important outcome, i.e. an efficient bundle of acts of legislation. In this paper we considered the case of two-parties (or two-coalition) systems. In such a situation, there is always a party (or a coalition) in a position to command an absolute majority in Parliament. This kind of system is very effective in delivering decisions, so that this ability makes up for the failings of representation and party discipline. In fact, we have shown that one can improve upon that outcome. This is possible when independent legislators, randomly selected from the relevant constituency, are allowed to sit in Parliament without any obligation to join a party and accept its discipline.

The main argument was based upon the costs a representative democracy typically entails. Those costs emerge from the failure of the relationship between constituents and political representatives. It typically happens that political representatives cease to be faithful agents of their constituents and turn into agents of their parties. This is the contemporary expression of a long-standing power dilemma: factionalism. As extensively illustrated in the paper, this shift in the agency relationship deprives the representative system of its ability to deliver decisions that command a large consensus in the population of constituents. For one can expect that the system delivers unpopular decisions just as easily as it can deliver popular ones. Its cost, therefore, is the forgone benefit of not delivering just popular decisions.

Factionalism is not a new development. Throughout history it has taken up various forms and has worked through various mechanisms [27]. The first instance of sortition as a way to organize power goes back to ancient Athens, as is very well known. It was Kleisthenes who put ten tribes in place of the long-established four tribes. According to Aristotle [28], Kleisthenes broke up the Athenian region into thirty groups called trittyes: ten from the city, ten from the coast and ten from the inland. Coming from the same region, each trittys was highly homogeneous. Therefore, the trittyes were very likely to turn into opposing factions, as they represented diverse interests. Turning to lot avoided this occurrence, as each tribe was made up of three different trittyes. In turn, those ten tribes would make up the population of the 500 members of the Boule. Fifty representatives would be selected from each tribe.

Much later, between the late Middle Ages and the Renaissance, sortition became very popular in Italian city-states. The '*Brevia*' and the '*Scrutinio e tratta*' were the two main instances. The '*Brevia*' spread across Northern Italy between the twelfth and the thirteenth century and survived, in the version that got established in Venice, until the beginning of the nineteenth century [29]. The '*Scrutinio e tratta*' was developed in Florence at the beginning of the fourteenth century and survived for one and a half century [30].

There is evidence that in the 'Scrutinio' sortition was precisely designed to overcome opposition among existing factions.[3] It is interesting to note that, alongside that scheme of the 'Scrutinio', it was ruled out that the relatives of those who had been selected through sortition for a public office could be selected themselves for the same office. It was just another way to contrast the same enemy: factionalism. Similar evidence exists in the case of '*Brevia*', introduced in 1233 in the city of Parma; there, sortition is clearly presented as a means to avoid contentions among factions.[4] Interestingly, the '*Brevia*' and the '*Scrutinio*' mirrored each other. In the '*Brevia*' sortition came at the beginning of the selection procedure. Once selected the electors would go on to vote or nominate their representatives. The '*Scrutinio*' would work the other way round. First, the pool of representatives would be elected. Then, starting from that pool, public offices would be allocated by lot. Using sortition interchangeably, at different stages of the selection procedure, shows that the purpose of sortition is really fighting factionalism, just like ancient Greece. Furthermore, it shows that random procedures can coexist with teleological ones. Sortition would add to the conscious choice of representatives something that choice could not offer. Sortition is neutral, as it rules out persuasion. It also implies no responsibility, as it rules out mandate. Both elements could prove to be useful.

It must be stressed that in the cases mentioned before sortition by lot – and thus the unpredictability of outcomes – has in actual fact contributed to reduce factionalism. The frequent turnover of members of bodies like the Athenian Boule or the Florentine Signoria made it very difficult for individual members to establish a reputation. Quite clearly this made the discussions and negotiations necessary to reach collective agreements rather lengthy and difficult. In economic parlance this implied high transaction costs. The practice of log-rolling, i.e. the exchange of favours, was as a result of such difficulties unlikely to develop, both within and among factions. Thus, by weakening the likely bonds to be established within each political group, sortition has done a good job in contrasting factionalism

Along the same line, it has been shown in this paper a possible way of adopting sortition in order to repair the fallacy of factionalism and minimize the cost of the modern representative democracy. The logic underlying this endeavour is preserving the ability of representative democracy to deliver decisions while increasing as much as possible their net contribution to welfare. The way to do so is shifting the balance of decision-making towards the positive side. Letting a number of independent legislators, drawn at random from common citizens, sit in Parliament with the same prerogatives of the other legislators, has an outstanding advantage: it improves the net benefits of the parliamentary outcomes.

By gradually allowing more and more independent legislators in Parliament, the detrimental effects of party discipline are curtailed and the beneficial effects of voting according to one's preferences are enlarged. There comes a point where the ability of a party to make its members vote for whatever comes from within loses its strength, thus magnifying the role of those legislators who are not subjected to any imposition. However, one should not think that the larger the number of independent legislators, the better. Party discipline has its merits and should not be discarded altogether. As extensively shown in the previous sections, if only independent legislators sat in Parliament, hardly any decision would be taken, whether good or bad. The model of Parliament illustrated in this paper has precisely shown that a virtuous combination of party discipline and freedom of choice is possible.

We are fully aware that, in usual conditions, a system that largely reduces the parties' power would hardly be adopted. The elected legislators have no advantage to implement a change that substantially reduces both the possibility to be reelected and, therefore, their power. However, our proposal should not have sounded so abstruse in a number of real

[3] "*Dappoich'è Fiorentini ebbono novelle della morte del duca, ebbono più consigli e ragionamenti e avvisi, come dovessono riformare la città di reggimento e signoria per modo comune, acciocché si levassono le sette tra' cittadini*": Villani (1845), III, p.10.

[4] "*Capitulum ad evitandum quod aliquis qui non sit de consilio generali debeat stare ad sortes recipiendas, et ad evitandum contentiones super hoc*": Statuta Communis Parmae (1855), II, p. 39. In fact, beyond factions, sortition was also intended to reduce corruption and violence: see Wolfson (1899), p. 12.

recent experiences. As a matter of fact, sortition tools are gradually making their way into the contemporary political scenario [31]. It is the case of the *Irish Constitutional Convention* of 2013 and, more recently, the cases of both the *Observatorio de la Ciudad* of Madrid (2018) and the permanent authority that will support the regional Parliament of German-speaking Community of Belgium (2019) [32].

Another possible motivation for implementing a mixed parliament could be related with the problem of the increasing level of abstention which is affecting the modern democracies in the last decades. In this respect, for example, one could think to provide with two alternative options to those citizens who are typically oriented towards abstention: each of them, going to the polls during the election day, could choose whether to vote for a party candidate or enrol in a list for a sortition. Then, after the elections, a percentage of seats proportional to the area of abstention would be reserved to randomly selected citizens picked out from the list of candidates. Of course the remaining seats would be assigned to candidates of the parties in a proportion established through voting.

It is likely that such a procedure will give rise to a parliament without any absolute majority, and this could be considered dangerous in some parliamentary systems (like for example the Italian one), since it could make difficult to have a stable Government. But, as we show in this paper, the absence of an absolute majority, because of independent legislators, is precisely what improves the efficiency of the parliament. Finally, to protect the independent legislators from being captured by the existing parties, we could also think of a system of rotation, so that new independent legislators would be selected at random (from the original sortition list decided by the abstention level) for each single parliamentary session, devoted to a specific issue.

In any case, as a matter of fact, we are not concerned here with details of any dynamic path leading from one composition of Parliament to another. The theoretical exercise we performed is interesting *per se*: it shows that one of the possible reasons of parliamentary inefficiencies is party discipline. Sortition is beneficial because, by contrasting such a discipline, hinders factionalism. Just like in old Italian city-states, when sortition could be effectively combined with more traditional ways of selecting public officers to tame the undesirable implications of factionalism.

CRediT authorship contribution statement

Maurizio Caserta: Conceptualization, Investigation, Writing - original draft, Writing - review & editing. **Alessandro Pluchino:** Conceptualization, Formal analysis, Investigation, Writing - original draft, Writing - review & editing. **Andrea Rapisarda:** Conceptualization, Investigation, Writing - original draft, Writing - review & editing. **Salvatore Spagano:** Conceptualization, Investigation, Writing - original draft, Writing - review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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[32] For more details about real experiments and experiences around the world see the website: www.oderal.org.